PG – 373

I Semester M.Sc. Degree Examination, January/February 2014
(RNS-Y2K11 Scheme)
MATHEMATICS
M 104 : Ordinary Differential Equations

Time : 3 Hours

Max. Marks : 80

Instructions : 1) *Answer* ***any five full*** *questions, choosing atleast* ***two*** *from* ***each*** *Part.*

2) ***All full*** *questions carry* ***equal*** *marks.*

PART – A

1. a) A set of n solutions $\{\phi_j(x), j = 1 \text{ to } n\}$ of $L_n y = 0$ in the interval I forms a fundamental set iff

$W\{\phi_j(x), j = 1 \text{ to } n\} \neq 0$ for all $x \in I$. 8

b) Find the Wronskian of the independent solutions of

$$\frac{d^5y}{dx^5} - \frac{d^4y}{dx^4} - \frac{dy}{dx} + y = 0, x \in (-\infty, \infty).$$ 8

2. a) With usual notations, prove that

$$g(x) L_n f(x) - f(x) L_n^* g(x) = \frac{d}{dx}[f, g](x).$$

Further show that $[f, g](x) = -[g, f](x)$ if L_n is self-adjoint. 10

b) Given f(x) and g(x) are two functions having n continuous derivatives on [a, b]. Show that f(x) is a solution of $L_n y = 0$ on [a, b] iff it is a solution of $[y, g](x) = C$, where g(x) is a solution of $L_n^* y = 0$ and C is a constant. 6

P.T.O.

3. a) If $\phi_1(x)$ and $\phi_2(x)$ are linearly independent solutions of a self-adjoint differential equation on some interval then show that between two consecutive zeros of one solution, there is a unique zero of another solution. 7

b) Show that the equation

$$y'' + \left\{ \frac{1}{4x^2} + \frac{k}{(x \log x)^2} \right\} y = 0 \quad (0 < X \leq x < \infty), \text{ where k is a constant, is}$$

oscillatory if $k > \frac{1}{4}$ and non-oscillatory if $k \leq \frac{1}{4}$. 9

4. a) Define a self-adjoint eigen value problem. For such a problem, show that

i) the eigen values are real and

ii) the eigen functions corresponding to distinct eigen values are orthogonal over the relevant interval. 12

b) Construct Green's function for the problem 4

$$y'' + \lambda y = e^x; y(0) = 0 = y(\pi).$$

PART – B

5. a) Find the ordinary, regular and irregular points, if any of the differential equation :

i) $xy'' + (1-x)y' + \alpha y = 0$ 6

ii) $x^2 y'' - xy' + (x^2 - x^2) y = 0$. 5

b) Prove the orthogonality of the Hermite polynomials. 5

c) Show that $\frac{1}{1-t} \cdot e^{\frac{-xt}{1-t}}$ is the generating function for Laguerve polynomials. 5

6. a) Derive the following recurrence relations for Tchebyshev polynomials :

i) $T_{n+1}(x) - 2x\, T_n(x) + T_{n-1}(x) = 0$ 8

ii) $(1-x^2) T_n'(x) + nx\, T_n(x) - n\, T_{n-1}(x) = 0$.

b) Using Frobenius method obtain a solution of the Gauss hypergeometric equation about any one regular singular point. 8

7. a) Find the fundamental matrix solution of the linear system :

$$\frac{dx_1}{dt} = 4x_1 - x_2 \; ; \; \frac{dx_2}{dt} = x_1 + 2x_2$$

Hence find the general solution. 6

b) Find the nature and stability of the critical points of the system :

i) $\frac{d^2x}{dt^2} = 4x^3 - 4x$

ii) $\frac{dx}{dt} = x + 4y - x^2; \frac{dy}{dt} = 6x - y + 2xy$. 10

8. Apply Liapunov method to determine the stability of the critical point (0, 0) of the systems :

a) $\frac{dx}{dt} = y - 2x^3; \frac{dy}{dt} = -2x - 3y^3$

b) $\frac{dx}{dt} = -x^3 - 8xy^2; \frac{dy}{dt} = -2x^2y + 9y^3$. (8+8)